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A Concise History of the Modern Calculus in Mathematics

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Mathematics without History is mathematics stripped of its greatness. Like the other arts, mathematics is also one of the supreme arts of civilization as it derives its grandeur from the fact that it is a human creation. Calculus is a body of calculations or reasoning which are associated with a certain concept. The basic idea of present day calculus had been fermenting in intellectual circles in the seventeenth century.

The word Calculus was derived in mid-17th century from the Latin word Calculus literally meaning, "small pebble" as used for counting on abacus. Calculus is a very important branch of mathematics that deals with a particular method of calculation and mathematical study of continuous change. It involves in it applications of science, engineering and economics. It majorly has two branches namely: differential calculus that concerns rates of change and slopes of curve and integral calculus that concerns with gathering of quantities and the areas under and between curves. Both the branches make use of fundamental concepts of convergence of infinite sequences and series to a well-defined limit. In differential calculus, that concept is the derivative which is one of the fundamental ideas in complete mathematics and it can also be considered as a cornerstone of modern scientific thought. Calculus has historically been known as 'the calculus of infinitesimals'. It is a history of mathematics that emphasizes on limits, functions, derivatives, integrals and as well as on infinite series.

Emergence of Calculus was effectively demanded by the philosophical spirits of the times. Natural philosophers had long believed that the universe was constructed according to understandable mathematical principles, although they disagreed about just what these principles were and how they might be formulated. The early astronomers announced that heavenly bodies move in circular orbits around the earth as center and later that earth itself must be a perfect sphere to reflect the divine hand of its Creator. But all these assertions are now known to be false. In 1612, Kepler explained these motions with scientific formulas. Galileo announced that the distance travelled by a heavy body falling from rest is proportional to the square of the elapsed time, and in 1657 Fermat asserted that light moves along those paths that minimize the time of travel. The question was whether such laws can be formulated and justified mathematically, and what kind of mathematics would be appropriate to describe these phenomenon.

Bhaskaracharya was an astronomer and an Indian mathematician of 12th century A.D. He was born near Vijjadavida (Bijapur in modern Karnataka) and lived between 1114-1185 A.D. He was the head of the astronomical observatory at Ujjain which was the leading mathematical centre of Ancient India. He learned mathematics from his father Maheswara who was an astrologer. The main work of Bhaskaracharya was Siddhanta Shiromani, which means "Crown of treastises" in Sanskrit. It is divided into four parts called Lilavati, which means beautiful woman and it was named after his daughter lilavati Bijaganita, Grahaganita (mathematics of planets) and Goladhyaya (study of planets and spheres) respectively. He also wrote another treatise named Karna Kautoohala. These four sections deal with arithmetic, algebra, mathematics of the planets and spheres respectively. Bhaskara's work on Calculus predates Newton and Leibniz by over half a millennium. He is particularly known in the discovery of the principles of differential calculus and its application to astronomical problems and computations. Though Newton and Leibniz have been credited with differential and integral Calculus, there is also a strong evidence to suggest that Bhaskara was a pioneer in some of the principles of differential calculus.

He was perhaps the first to conceive the differential coefficient and differential calculus. Bhaskara was the first to introduce the concepts of infinity which states “If any finite number is divided by zero the result is infinity”. Also the fact that if any Finite number is added to Infinity then the sum is infinity. He is known for his calculation of the time required (365.2588 days) by the earth to orbit the sun which differs from the modern day calculation of 365.2563 days, by just 3.5 minutes. The law of gravitation had been proved by Bhaskara 500 years before it was discovered by Newton.

The controversy has been on over who invented calculus, the German mathematician and logician Gottfried Wilhelm Leibniz whose period is considered from 1st July,1646 to 14th November,1716 or the English mathematician and physicist Sir Isaac Newton whose period is from 25th December , 1642 to 20th March ,1726/27 . This calculus controversy often referred to as Prioritätsstreit, meaning "priority dispute" had been a major intellectual controversy, one that began simmering in 1699 and broke out in full force in 1711. The product rule and chain rule and other concept such as higher derivatives, Taylor series were all introduced by Isaac Newton which he used to solve problems of mathematical physics. Newton invented calculus while he was doing research in physics and geometry. He used calculus to solve the problem of planetary motion, the oblateness of the earth, the shape of the surface of a rotating fluid, and many other problems which are described in his work in Principia Mathematica. In his other works he developed series expansions for functions but he did not publish all his discoveries as at that time infinitesimal methods were still infamous. Newton derived his results first in 1666, at the age of 23, but did not publish it except decades later which he called it as the method of fluxions and fluents and called his calculus the science of fluxions. The book was completed in 1671, and got published in 1736.

Gottfried Leibniz started working on calculus in 1674 and in 1684 published his first paper Nova Methodus pro Maximis et Minimis. He provided a clear set of rules for working with infinitesimal quantities for the computation of higher derivatives in differential and integral forms. Unlike Newton, Leibniz arranged these ideas into a true calculus of infinitesimals and paid a lot of attention to formalize the concepts of calculus by determining appropriate symbols for concepts. Both Leibniz and Newton are credited with the invention of calculus. Newton became the first to apply calculus to general physics and Leibniz developed annotation used in calculus today. Leibniz was earlier accused of copyright infringement by Newton. This was because many of Newton’s colleagues had connections with Leibniz and some of Newton’s unpublished manuscripts may have found a way to Leibniz’s hand. Newton claimed that Leibniz has stolen ideas from his unpublished notes. But after much controversy and discussion Leibniz is now regarded as an independent inventor of calculus and is known as the father of Calculus. When both the mathematicians published their works there was a great debate as to who deserved the credit, which divided the English speaking mathematicians from the continental European ones. Newton also had powerful allies working in his favor to support his publication Principia Mathematica, the publication that made him a scientific celebrity. The genius of Newton and Leibnitz centered not so much on the discovery of those ideas as on their systematization.

A careful examination of the findings of Newton and Leibniz was done that showed that they arrived at their conclusions independently. Leibniz started his work on calculus first with integration and Newton started his work first with differentiation. That is why both the mathematicians are known to develop calculus and contribute to this field independently around the same time. Gottfried Leibniz was indeed a remarkable man and a phenomenal mathematician. During his lifetime (between 1646 and 1716), he discovered and developed the most important and salient mathematical theories. In addition to this, he postulated many theories on mathematics, physics and human philosophy.

Now, after discussing about who discovered calculus, there has always been a question in people's mind that what was the need of developing such a terrible, confusing, jumbled mess of illogical expressions and rules known as mathematics and calculus, that many people just give up trying to avoid at some point. Nevertheless, many students of mathematics persist and study through years of algebra and arithmetic to find themselves facing a very different level of difficulty of maths: Calculus. Initially, mathematics emerged to solve problems and predict the outcomes of every action in everyday life, and as time passed humans became more and more interested in knowing how the world worked, but they were faced with the limitations of their existing mathematical theories at that time. That's why many mathematicians throughout the world worked to create new and better models of nature that would lead to advanced mathematics. The emergence of calculus stimulated an immense and energetic further development. On the English side, Taylor and Maclaurin, on the continent the Basel mathematicians, first the brothers Bernoulli, then the prodigious Euler, later the Frenchman D'Alembert and the Italian Lagrange contributed greatly to the development. Moreover the other parts which were connected to mathematics and closely related to calculus soon originated like the theory of differential equations, the calculus of variations, differential geometry, etc.

Although Newton and Leibniz had rather reasonable ideas about the fundamentals of the new Calculus, the very rapid development of Calculus resulted in the fundamentals being neglected or treated unsatisfactorily. For example D'Alembert stressed on using limit as the fundamental of Calculus. But it was finally Cauchy, who could give a systematic and consistent explanation to the theories of Calculus, forming a solid basis for it. By the second half of the 1700s, it was generally accepted that without logical underpinnings, Calculus would be limited. Cauchy developed an acceptable theory of limits, and in doing so removed much doubt about the logical validity of Calculus. Cauchy wrote a treatise on integrals in 1814 that is considered a classic and in 1816 his paper on wave propagation in liquids won a prize from the French Academy. Cauchy did not formulate the ϵ-δ

definition of limit that we use today, but instead formulated a purely arithmetical definition. His definition of limit appeared in his monumental treatise, Cours d’ Analyse de l’ Ecole Royale Polytechnique which came in 1821.

John Napier who is best known today as the inventor of logarithms was a Scottish land owner who was the Isaac Asimov of his day. He was the one who envisioned the tank, the machine gun and the submarine. He even predicted the end of the world would occur between 1688 and 1700. Napier’s Logarithms are not identical, to the logarithms we use today. Napier chose to use 1-10-7 as his given number which was then multiplied by 107. If N=107 (1-1/107)L then L is Napier’s Logarithm of the number N that is L = log N means N = 107(1-10-7)L . Napier’s 1614 paper on Logarithms was read by a true mathematician Henry Briggs (1561-1630) and together they decided that base 10 made a lot more sense. In the year in which Napier died, Briggs published a table of common Logarithms (base10) which was a major accomplishment of that time. In this paper he used the words “mantissa” and “Characteristic” which we use even today in studying the values of Logarithms.

Precursors of Calculus

Ancient

In early ages, traces have been found that show that ancient period introduced some of the ideas that led to integral calculus, but at that time it was not developed in an accurate and systematic way. Calculations of volume and area, a part of integral calculus can be found in the Egyptian Moscow papyrus in 1820 BC. At that time there was no indication of any formal method being used for calculation but only simple instructions that lacked major components. Babylonians may have discovered the trapezoidal rule while doing astronomical observations of Jupiter. Method of exhaustion was first used in age of Greek mathematics by Exodus in 408-355 BC that indicated the concept of limit to calculate volumes and areas. Between 287-212 BC Archimedes further developed the idea of calculus by inventing heuristics which resembles the method of integral calculus. Later in the 3rd century AD the method of exhaustion was discovered independently in China by Liu Hui to find the area of a circle.

Archimedes was the first person to find the tangent to a curve other than a circle. The developers of the calculus such as Isaac Barrow and Johann Bernoulli were meticulous students of Archimedes that assisted him. The Bernoulli family of Switzerland produced at least eight noted mathematicians over three generations. Two Brothers, Jacob (1654- 1705) and Johann (1667-1748), were bitter rivals .These brothers were extremely influential advocates of the newly born calculus. Johann was the most prolific of the clan and was responsible for the discovery of L' Hopital's Rule, Bernoulli numbers, Bernoulli polynomials ,the lemniscate of Bernoulli ,the Bernoulli equation ,the Bernoulli Theorem , and the Bernoulli Distribution . He did a great deal of work with differential equations but at the same time he was a jealous person and cantankerous so much so that he had rivalry with his own son whom he threw out of his house for winning an award he had expected to win himself.

Medieval

The ground work for much of the mathematics that we have today and certainly which is a necessity for Calculus is the development of analytic Geometry by Descartes and Pierre de Fermat. Fermat was a lawyer by profession who used to do mathematics in his spare time. He wrote well over 3,000 mathematical papers and notes. Fermat developed a general procedure for finding tangent lines that is a precursor to the methods of Newton and Leibniz. Isaac Newton who invented calculus at the same time as Leibniz considered Fermat as "one of the giants" on whose shoulders he stood. Descartes' ideas for analytic geometry were published in 1637 as one of the three appendices to his discourse on the method "of reasoning will and seeking truth in the sciences". In that same year, Fermat wrote an essay entitled "Introduction to plane and solid Loci" in which he laid the foundations for analytic geometry. Though Fermat's paper was more complete and systematic, but yet Descartes' paper was published first and therefore he is credited with the discovery of analytic geometry. Today's Cartesian coordinate system and Cartesian Geometry is attributed to Descartes' discovery. The analytic geometry we know today is studied from two view point which were given by Descartes and Fermat. Descartes' viewpoint was that if "Given a curve, describe it by an equation" whereas Fermat talked about "given an equation, describe it by a curve". Pierre de Fermat also obtained the first method for differentiating polynomials, but his real was number theory. Fermat's last theorem is his most famous problem. He used to write anywhere where he could find space while eating or drinking or doing any other daily activities. He wrote in the major of a text: "To divide a cube into two cubes, a fourth power, or in general, any power whatever above the second, into powers of the same denominations, is impossible, and I have assuredly found on admirable proof of this, but the margin is narrow to contain it".

Calculus and India

It has always been a matter of controversy whether or not calculus was invented in India. Many researches have been done to find the actual truth behind the long lived history of calculus in India. Once two British researchers namely Joseph and Almeida challenged the evidence and conventional history of mathematics in India, that whether one of the main concepts of calculus, the infinite series was developed by the Indian mathematicians? They spent three years in research through ancient Indian and Vatican texts. They believe that the scientific knowledge of calculus has travelled to Western Europe from India itself. The missionary priests in India in middle 16th century learned the local languages and sent the scrupulous reports back to the Europe. Mathematical historians have long known about the contribution of Kerala's mathematician Madhava towards calculus, but it has not yet been established that how his contributions might have influenced Newton or Leibniz in their discoveries and inventions. Historian of mathematics of University of Manchester, George Gheverghese Joseph who conducted the research with Almeida of the University of Exeter said, "The notation is quite different, but it's very easy to recognize the series as we understand it today." Bhaskaracharya II also made many contributions and remarkable improvements upon existing knowledge of calculus and worked with what is today called as derivatives and integrals. He even had an initial notion of infinitesimals. In the 14th century the Kerala mathematicians, starting with Madhava of Sangamagrama, developed some amazing calculus thereby stating components of calculus. He discovered power series, gave expansions of trigonometric sine, cosine and arc tangent functions.

By now it has been proved to be a well- established fact that calculus and the infinite series originated in India across thousands of years ago, starting from 5th CE. Although, the Indian mathematicians at that time were not able to show a connection between the two themes of calculus: derivative and integral but however they have a big hand in its invention. Calculus was needed for agricultural works and overseas trade, the two key sources of Indian wealth at that time. However, Europeans earlier did not understand the Indian arithmetic, that is, the Indian methods of summing up infinite series using 'non-Archimedean' arithmetic, a kind of formal philosophy which in today's world is known as zeroism.

Transmission of Calculus to Europe

The discovery of infinite series in India was followed by the introduction of the same series in Europe and since both the series were connected there was a question that if or not these identical series were rediscovered independently. The obligation to proof actually lied with the people who claimed that both the series were independently rediscovered in Europe, when Europe was still struggling to learn addition and subtraction without using abacus.

Europeans were backward in navigation and navigation required accurate trigonometric values with accurate trigonometric tables. Hence, European governments offered huge incentives and prizes for a solution in navigation problems and difficulties. Therefore, the Jesuits turned their Cochin College into an institution for bulk translation of Indian texts to European text in 1550. They sent these translated texts back to Europe on the Toledo model. The problem arose as the Europeans did not have a great deal of knowledge about trigonometry and its operations to determine the size of the globe. Thus, in the later 16th c. and early 17th c. Indian texts started appearing in the European continent to solve the problems related to latitude and loxodromes. The Jesuits had access to all the information they needed to achieve their goals, because of the support of the king and local community of Syrian Christians until 1600.

There are other evidences that show that Europeans work was similar to the Indians at that time. Tycho Brahe who gave 'Tychonic model' , his work was similar to that of Nilakantha , "Julian" day numbers similar to Ahargana, Christoph Clavius's trigonometric values interpose version of Indian values, Kepler's findings were similar to Parameswaran's observations, Leibniz's and Newton's works (Leibniz series and Sine series respectively) are among some other examples.

Pioneers of Calculus of the Modern Era

Since the time of Leibniz and Newton, many mathematicians have contributed towards the development of calculus. In the 17th century, European mathematicians like Isaac Barrow, Rene Descartes, Pierre de Fermat, Blaise Pascal, John Wallis among some others discussed the idea of a derivative. One of the first complete works on calculus was written by a female mathematician Maria Gaetana Agnesi when her classic calculus text book was published in 1748. The work was based on both the subjects of infinitesimal and integral calculus. Her first publication was at the age of nine when she wrote a Latin discourse defending higher education for women. She is primarily remembered for a curve defined by the equation y=a^3/(x^2+a^2) where a is a positive constant. The curve was named Versiera meaning to turn by Agnesi but John Colson, an Englishman who translated her work misunderstood this word and called it Avversiera, which means “wife of the devil” in Italian. Therefore, the curve has ever since been called the “witch of Agnesi”.To be called by this name was unfortunate because Colson wanted Agnesi’s work to serve as a model for budding young mathematicians. Pierre Fermat developed a method known as adequality method for determining maxima, minima, and tangents to various types of curves that was closely related to differentiation. He compiled this method in his works Methodus ad disquirendam maximam et minima and in De tangentibus linearum curvarum.

Cavalieri's principle was established in the 5th Century AD to find the volume of a sphere. In the 1630s and 1640s, Cavalieri developed his method of indivisibles on the integral side, providing a more modern form of the ancient Greek method of exhaustion. He also computed Cavalieri's quadrature formula, to find the area under the curves xn of higher degree, which was used earlier only for computing the area under a parabola, by Archimedes. Torricelli carried on this work to other curves such as the cycloid. He extended his work so much so that after some time a generalized formula was made for fractional and negative powers by Wallis in 1656. Fermat in a discourse in year 1659 is attributed to an original technique for evaluating the integral of any power function directly. He also obtained a technique for finding out the centers of gravity of various planes and solid figures, which influenced further work to develop in quadrature. James Gregory was influenced by Fermat's contributions to both tangency and to quadrature. So, in the middle of 17th century he then proved a restricted and limited version of the second fundamental theorem of calculus. The full proof of the fundamental theorem of calculus was first given by Isaac Barrow.

Newton and Leibniz, while independently developing on this work, advanced the surrounding theory of infinitesimal calculus in the late 17th century. Leibniz also did a great deal of work and developed consistent and useful notation and concepts of it. On the other hand, Newton provided some of the most important applications to physics, and especially of integral calculus. The first ever proof of the Rolle's theorem was given by a mathematician known as Michel Rolle in 1691 using methods developed by the Dutch mathematician Johann van Waveren Hudde. The mean value theorem in it was indeed stated by Bernard Bolzano and Augustin-Louis Cauchy (1789–1857) after the discovery of modern calculus. Important contributions were also made by Barrow, Huygens, and many others in the field of physics as well as in mathematics, especially in the line of calculus.

Many mathematicians like Maclaurin tried to prove and show the advantages and positive aspects of using infinitesimals in calculus, but it was not until 150 years later when, a way was finally found by Cauchy and Weierstrass to avoid basic notions of infinitesimally small quantities. Cauchy introduced the concept of continuity for a function defined on an interval in the same way as Bolzano. In his book, Cauchy even points out that the continuity of many functions is easily verified. The formal definition of limit that we use today was first given by Weierstrass ,who was a German secondary school teacher. David Burton describes Weierstrass as the father of Modern Analysis.

Importance of Calculus

Today, calculus is a valuable tool in mainstream economics. Besides the significant aspect that this part of mathematics helps in development of an analytical mathematical thinking, calculus proves its effectiveness by solving real life and practical problems.

In fact, calculus can be applied in many things in a lot of ways and applications. Among the disciplines that utilize calculus include physics, engineering, economics, statistics, and medicine. It is used to create mathematical models in order to arrive to a favorable solution. Applications of differential calculus include computations that involves velocity and acceleration, the slope of a curve, optimization, area, volume, arc length, center of mass, work, and pressure. More advanced applications includes calculation of power series and Fourier series.

Calculus is also used to gain an accurate understanding of the nature of space, time, and motion. For years and years, mathematicians and philosophers struggled with paradoxes that involved sums of infinitely many numbers or division by zero. These questions many times arise in the study of motion and area. Calculus provides tools, especially the limit and the infinite series, which resolve the paradoxes. In the history of sciences, the invention of calculus has been one of the greatest inventions of all times. As stated by John von Neumann, "The calculus was the first achievement of modern mathematics and it is difficult to overestimate its importance. I think it defines more unequivocally than anything else the inception of modern mathematics, and the system of mathematical analysis, which is its logical development, still constitutes the greatest technical advance in exact thinking." Calculus has provided a mathematical language that, by means of the derivative, or rates of change can be used to characterize various physical processes and by means of the integral it can be shown how the macroscopic entities such as area or distance can emerge from properly assembled microscopic elements. Finally, the ability to relate the results of limiting arguments by simple algebraic formulas consents the correct use of calculus while retaining skepticism regarding its foundations. This has helped applications to prosper while mathematicians have sought an appropriate axiomatic basis for themselves.

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